<Symmetric matrices>

① eigenvectors → perpendicular
② eigenvalue → REAL.

Usual case $A = S\Lambda S^{-1}$

Symmetric case $A = Q\Lambda Q^{-1}$
$= Q\Lambda Q^{T}$

(~ principal axes)

∵ Why real eigenvalues? $\overline{a+ib} = a-ib$

$Ax = \lambda x \Rightarrow$ always $\bar{A}\bar{x} = \bar{\lambda}\bar{x}$ ($A\bar{x}$)

$\bar{x}^T A x = \lambda \bar{x}^T x$ $\Rightarrow \bar{x}^T A^T = \bar{x}^T \bar{\lambda} \Rightarrow \bar{x}^T A x = \bar{\lambda}\bar{x}^T x$ ($A^T = A$)

$\lambda \bar{x}^T x = \bar{\lambda}\bar{x}^T x$ ∴ $\lambda = \bar{\lambda}$ is real!

* Good matrices

Real λ's
perpendicular x's

∴ $A = A^T$ → $A = Q\Lambda Q^T$

~~A=A~~.

$$= \begin{bmatrix} q_1 & q_2 & \cdots \end{bmatrix}\begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{bmatrix}\begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_n^T \end{bmatrix}$$

$= \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots$

Every symm. matrix is a combination of perpendicular projection matrices.

Signs of pivots same as signs of λ's

# of pivots (pos.) = # positive λ's

<positive definite (symmetric) matrix> 23

= Symmetric matrix with all eigenvalues are positive.

(= all pivots are positive)

→ determinant → positive.
↑ sub

<Complex vectors>

Fourier transform → n^2 multiplications
~~FF~~ Fast Fourier trans. → $n\log_2 n$ 〃

$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$ Find length in C^n → $z^T z$ no good
→ $\bar{z}^T z$ is good

$\begin{bmatrix} 1 & -i \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = 1+1 = 2.$

$z^H z$

· inner product $\{y^H x\}$ Hermitian.

Symmetric $A^T = A$ no good if A complex.

Hermitian $A^H = A$.

Orthogonal complex matrix $\bar{Q}^T Q = I$
↳ Unitary matrix. $= Q^H Q$

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\ \vdots & & & & \\ 1 & w^{n-1} & w^{(n-1)^2} & & \end{bmatrix} \quad (F_n)_{ij} = w^{ij}$$

& $w^n = 1$. $w = e^{i\frac{2\pi}{n}} \to w^2 = e^{(i\frac{2\pi}{n})2}$

$F_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & \\ 1 & i & \\ 1 & -1 & \cdots \\ 1 & -i & \end{bmatrix}$ $F_4^H F_4 = I$. ∴ $F_4^{-1} = F_4^H$

cols orthonormal → Unitary

$$\begin{bmatrix} F_{64} \end{bmatrix}_{(64\times 64)} = \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} \text{Permutation} \end{bmatrix}$$

$$\begin{bmatrix} I & D \\ I & -D \end{bmatrix} \qquad \begin{bmatrix} 1 & & 1 & & \\ & 1 & & 1 & \\ & & \ddots & & \ddots \end{bmatrix}$$

$2(32)^2 + \cancel{64}\ \underset{32}{}$ multiplications..

$$D = \begin{bmatrix} 1 & & & \\ & w & & \\ & & w^2 & \\ & & & \ddots\ w^{31} \end{bmatrix}$$

$$[F_{64}] = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1 & 1 & \end{bmatrix}$$

$$\begin{bmatrix} I\ D & 0 \\ I\ -D & \\ 0 & I\ D \\ & I\ -D \end{bmatrix} \begin{bmatrix} F_{16} & & & \\ & F_{16} & & \\ & & F_{16} & \\ & & & F_{16} \end{bmatrix} \begin{bmatrix} P & \\ & P \end{bmatrix}$$

$2(2(16)^2 + 16) + 32$.

recursion

(why? use induction)

6×32

$\log_2 64 \qquad \frac{64}{2}$

ex) $n = 1024 = 2^{10}$

$n^2 \approx 1M$.

$\frac{1}{2} n \log_2 n = 5 \times 1000$

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* Positive Definite Matrixes (Tests)

- Tests for minimum, $(x^TAx > 0)$
- Ellipsoids in R^n

$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ ① $\lambda_1 > 0 \quad \lambda_2 > 0$

② $a > 0 \quad ac - b^2 > 0$

(1×1) det $\qquad (2\times2)$ det

③ pivots $a > 0 \quad \frac{ac-b^2}{a} > 0$.

④ $x^TAx > 0$

ex) $\begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix}$

Borderline of conditions)

= positive semidefinite $\lambda = 0, 20 \rightarrow$ pivot only 2

singular.

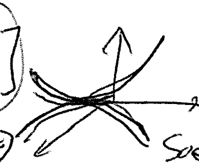
$$\underset{x^T}{[x_1\ x_2]} \underset{A}{\begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix}} \underset{x}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = 2x_1^2 + 12x_1x_2 + 18x_2^2 \overset{?}{>} 0$$

(barely failed!)

Graphs of $f(x,y) = x^TAx$

$= ax^2 + 2bxy + cy^2$

$\begin{bmatrix} 2 & 6 \\ 6 & 7 \end{bmatrix}$



Saddle point!

$\begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix}$

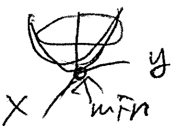
det = 4, trace = 22, positive..

$x^TAx > 0$ except at $x=0$

$f(x,y) = 2x^2 + 12xy + 20y^2$

$= 2(x+3y)^2 + 2y^2$

pivot① multiplier pivot②



1st deriv = 0.

calculus : min ~ $\frac{d^2u}{dx^2} > 0$

lin. alg : min ~ matrix of 2nd derivatives is positive Definite

$$\begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$2D \rightarrow \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

[3x3 example] $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ Pos. Def?

sub-det = 2, 3, 4

pivots = $2, \frac{3}{2}, \frac{4}{3}$

eigenvalues = $2-\sqrt{2}, 2, 2+\sqrt{2}$

$f = x^TAx = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 > 0$.

$Q\Lambda Q^T$

∴ Pos. Def! (if $f = 1 \rightarrow$ 3-principal axes.

A^TA is positive definite!
Similar matrices $A, B \to B = M^{-1}AM$
Jordan form

- Pos Def means
  $x^TAx > 0$ (except for $x=0$)
- If A, B are pos, Def,
  $x^T(A+B)x > 0$ so is $A+B$.
- Now A m by n with n indep cols $\Rightarrow$ rank n

$A^TA \to$ square, symmetric ok!
$\to$ pos.def?

$x^TA^TAx = (Ax)^T(Ax) = \|Ax\|^2 \geq 0$.

$n \times n$ A & B are similar ?
means for some M

$$B = M^{-1}AM$$

ex $S^{-1}AS = \Lambda$; A is similar to Λ

suppose $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$$\underset{M^{-1}}{\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \underset{M}{\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} -2 & -15 \\ 1 & 6 \end{bmatrix}$$

B similar to A.

what's similar? or what's same?
$\lambda_B = 3$ & 1!

"similar matrices have same eigenvalues"

$Ax = \lambda x$, $B = M^{-1}AM$
$(M^{-1}AM)M^{-1}x = \lambda M^{-1}x$
$B M^{-1}x = \lambda M^{-1}x$
$\Rightarrow$ eigenvector of $B = M^{-1}x$.

BAD case $\lambda_1 = \lambda_2 \to$ may not be diagonalizable ($\neq S$)

one small family has $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ $(= 4I)$ $M^{-1}(4I)M = 4I$.

big family includes $\begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix} \leftarrow$ Jordan form

More members of family

$\begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$ $\begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix}$ $\begin{bmatrix} a & m \\ m & 8-a \end{bmatrix}$

trace 8
det = 16

$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\lambda = 0, 0, 0, 0$
2 eigen vectors
$\dim N(A) = 2$.
not similar to $\to$ 2 missing.

$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Jordan block

$$J_i = \begin{bmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{bmatrix}$$

"Every square A is similar to a Jordan matrix J"

$$J = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & J_3 & \\ & & & J_d \end{bmatrix}$$

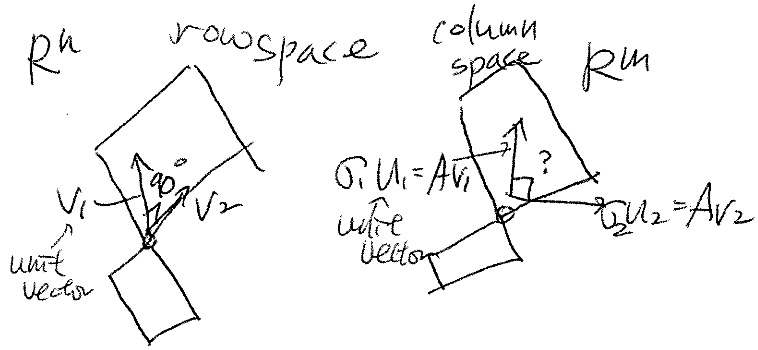
blocks = # eigenvectors.

Good; J is Λ.

Singular Value Decomposition (SVD)

$A = U\Sigma V^T$; Σ ~~orthogonal~~ diagonal
U, V orthogonal

Sym. pos def
$\to A = Q\Lambda Q^T$
~~$A = S\Lambda S^{-1}$~~



$$A[v_1 v_2 \cdots v_r] = [u_1 u_2 \cdots u_r]\begin{bmatrix}\sigma_1 & & \\ & \sigma_2 & \\ & & \ddots \, \sigma_r\end{bmatrix}$$

$$AV = U\Sigma$$

$A = \begin{bmatrix}4 & 4\\ -3 & 3\end{bmatrix}$ v_1, v_2 orthonormal in row space in R^2

u_1, u_2 in col space in R^2

$\sigma_1 > 0 \quad \sigma_2 > 0$

$$\begin{cases} Av_1 = \sigma_1 u_1 \\ Av_2 = \sigma_2 u_2 \end{cases}$$

$$A = U\Sigma V^{-1} = U\Sigma V^T$$

$$A^TA = V\Sigma^T \cancel{U^T U}\Sigma V^T$$

$$= V\begin{bmatrix}\sigma_1^2 & \\ & \ddots \, \sigma_r^2\end{bmatrix}V^T$$

ex1

$$A^TA = \begin{bmatrix}4 & -3\\ 4 & 3\end{bmatrix}\begin{bmatrix}4 & 4\\ -3 & 3\end{bmatrix} = \begin{bmatrix}25 & 7\\ 7 & 25\end{bmatrix}$$

eig. vec $= \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}$

$\xrightarrow{A^TA}$ $\lambda = 32\begin{bmatrix}1\\1\end{bmatrix}$ $\xrightarrow{A^TA}$ $\lambda = 18\begin{bmatrix}1\\-1\end{bmatrix}$

V!

$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \quad \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

$$\begin{bmatrix}4 & 4\\ -3 & 3\end{bmatrix} = \overset{U}{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}\overset{\Sigma}{\begin{bmatrix}\sqrt{32} & 0\\ 0 & \sqrt{18}\end{bmatrix}}\overset{V^T}{\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}}$$

U!

find u's u_1 u_2

$$AA^T = U\Sigma\cancel{V^TV}\Sigma^T U^T$$

$$= U\Sigma\Sigma^T U^T$$

$$AA^T = \begin{bmatrix}4 & 4\\ -3 & 3\end{bmatrix}\begin{bmatrix}4 & -3\\ 4 & 3\end{bmatrix} = \begin{bmatrix}32 & 0\\ 0 & 18\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$$

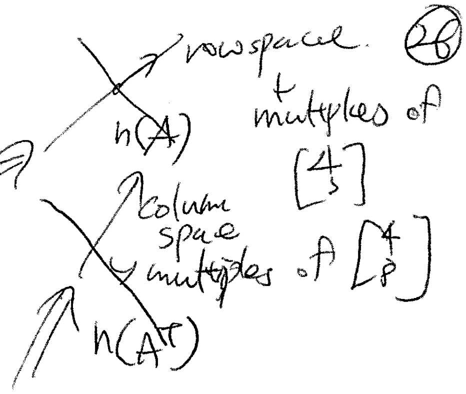
$\begin{bmatrix}1\\0\end{bmatrix} \xrightarrow{AA^T} \begin{bmatrix}1\\0\end{bmatrix} 32 \qquad \begin{bmatrix}0\\1\end{bmatrix} 18$

ex2 (26)

$A = \begin{bmatrix}4 & 3\\ 8 & 6\end{bmatrix}$

$v_1 = \begin{bmatrix}.8\\.6\end{bmatrix}$

$v_2 = \begin{bmatrix}.6\\-.8\end{bmatrix}$



$u_1 = \begin{bmatrix}1\\2\end{bmatrix}\frac{1}{\sqrt{5}}$

$$\underset{A}{\begin{bmatrix}4 & 3\\ 8 & 6\end{bmatrix}} = \underset{U}{\begin{bmatrix}1/\sqrt{5} & 2/\sqrt{5}\\ 2/\sqrt{5} & -1/\sqrt{5}\end{bmatrix}}\underset{\Sigma}{\begin{bmatrix}\sqrt{125} & 0\\ 0 & 0\end{bmatrix}}\begin{bmatrix}.8 & .6\\ .6 & -.8\end{bmatrix}$$

$$A^TA = \begin{bmatrix}4 & 8\\ 3 & 6\end{bmatrix}\begin{bmatrix}4 & 3\\ 8 & 6\end{bmatrix} = \begin{bmatrix}80 & 60\\ 60 & 45\end{bmatrix}$$

$0, 125$

$v_1 \cdots v_r \to$ orthonormal basis for rowspace

$u_1 \cdots u_r \to$ " " col space

$v_{r+1} \cdots v_n$ " " nullspace $N(A)$

$u_{r+1} \cdots u_m$ " " $n(A^T)$

and $Av_i = \sigma_i u_i$

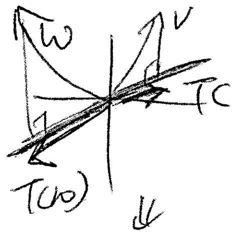
Linear Transformations T.

- without coordinates : no matrix
- with coordinates : matrix.

$$\begin{cases} T(v+w) = T(v) + T(w) \\ T(cv) = cT(v) \end{cases}$$

ex1 Projection

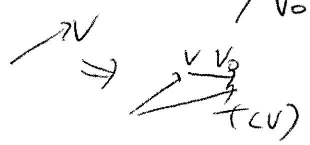
$T: R^2 \to R^2$



$\Downarrow$

linear

ex2 Shift whole plane by $\vec{v_0}$



$\Downarrow$

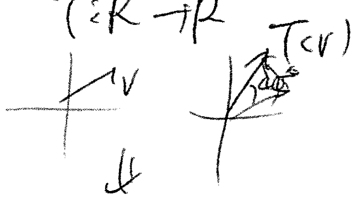
not linear.

$\Uparrow$

ex3 $T(v) = \|v\|$

ex4 Rotation by $45°$

$T: R^2 \to R^2$



$\Downarrow$

linear.

ex ~~4~~ matrix A!

$T(v) = Av \Rightarrow$ linear.

$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Start : suppose we have $T: R^3 \to R^2$

~~Example~~ : $T(v) = Av$ — $\boxed{2 \text{ by } 3 \text{ matrix}}$

output in R^2, input in R^3

"Information needed to know $T(v)$ for all inputs."

$\to T(v_1), T(v_2), \cdots T(v_n)$ for any basis $v_1 \cdots v_n$

Every $v = c_1v_1 + \cdots + c_nv_n$

know $T(v) = c_1T(v_1) + \cdots c_nT(v_n)$

Coordinates come from a basis

" of $v = c_1v_1 + \cdots c_nv_n$

$$v = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = \underbrace{3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{standard basis}}$$

* ~~cont~~ construct matrix A that represents lin tr T.

$T: R^n \to R^m$

* Choose basis $v_1 \dots v_n$ for inputs R^n.

" " $w_1, \dots w_m$ for outputs in R^m

Want matrix A.

$v = c_1v_1 + c_2v_2$

$T(v) = c_1T(v_1) + \cancel{c_2T(v_2)}$

$(c_1, c_2) \xrightarrow{T} (c_1, 0)$

$$\underset{A}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} \underset{\text{input coords}}{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}} = \underset{\text{output coord}}{\begin{bmatrix} c_1 \\ 0 \end{bmatrix}}$$

eigenvector basis leads to diagonal matrix Λ

◦ projecting onto $45°$ line

use standard $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = w_1$ $\quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = w_2$

◦ $P = \dfrac{aa^T}{a^Ta} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

* Rule to find A. Given bases $v_1 - v_n$, $w_1 - w_m$

1st col of A : write $T(v_1) = a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m$

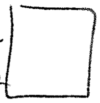
2nd col of A = $T(v_2) = a_{12}w_1 + \cdots a_{m2}w_m$

$\vdots$

$$A\begin{pmatrix}\text{input} \\ \text{coords}\end{pmatrix} = \begin{pmatrix}\text{output} \\ \text{coords}\end{pmatrix}$$

* $T = \dfrac{d}{dx}$ ~~(...)~~ (basis : polynomials)

Change of basis.
Compression of Images
Transformation $\longleftrightarrow$ matrix

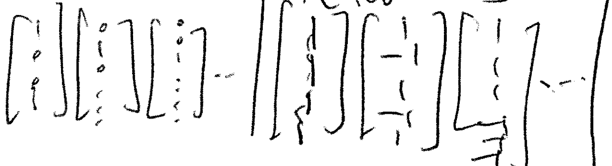
Image  512 × 512 $x \in R^n$. $n = (512)^2$

JPEG → change of basis.

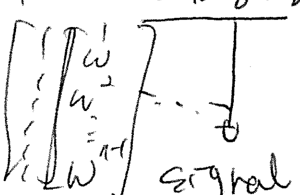
$x = \begin{bmatrix} x_i \end{bmatrix}$ $0 \le x_i \le 255$

512^2-dimension.

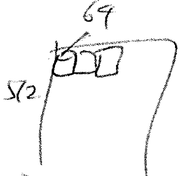
Standard basis | Better basis



Fourier basis 8 × 8



64, 512, 512

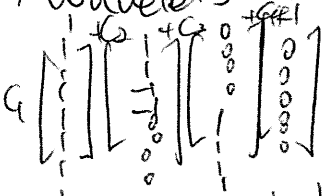


signal
(lossless) ↓ change
coeffs c.
(lossy) ↓ compress
$\hat{c}$ (many zeros)

$\hat{x} - \sum \hat{c}_i v_i$

video sequence of images
correlated.

* Wavelets R^8



coeffs c.

$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_8 \end{bmatrix}$ standard basis

$\rightarrow p = c_1 w_1 + \cdots c_8 w_8$

$\rightarrow p = Wc \rightarrow c = W^{-1}p$.

Good basis
① Fast FFT FWT.
② Few is enough.

Change of basis

Columns of W = new basis vectors

$[x]_{\text{old basis}} \longrightarrow [c]_{\text{new basis}}$ $x = Wc$

T with respect to $v_1 \sim v_8$, it has matrix A.
with respect to $w_1 \sim w_8$, it has matrix B.

similar $B = M^{-1}AM$

What is A? using basis $v_1 \sim v_8$
Know T completely from $T(v_1), T(v_2) \cdots T(v_8)$

Because every $x = c_1 v_1 + \cdots c_8 v_8$
$T(x) = c_1 T(v_1) + \cdots c_8 T(v_8)$

Write $T(v_1) = a_{11} v_1 + a_{21} v_2 + \cdots + a_{81} v_8$
$T(v_2) = a_{12} v_1 + a_{22} v_2 + \cdots$

$[A] = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{81} \\ a_{12} & \cdots & & \end{bmatrix}$

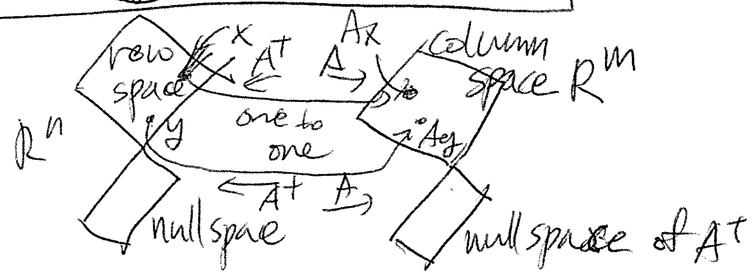
Eigenvector basis

$T(v_1) = \lambda_1 v_1$ What is A?

$A = \begin{bmatrix} \lambda_1 & 0 & & \\ 0 & \lambda_2 & & \\ \vdots & \vdots & \ddots & \\ & & & \lambda_n \end{bmatrix}$

1st input is v_1
its output is $\lambda_1 v_1$

4 Subspaces / pseudo-inverse / left, Right inverses



- 2-side inverse ⇒ inverse.

  ⇒ $AA^{-1} = I = A^{-1}A \Rightarrow r = m = n$ [Full rank]

- left inverse [Full column rank]

  $r = n < m$

  nullspace $= \{0\}$

  indep. cols.

  ~~row space~~

  ↳ zero or one solutions to $Ax = b$

  $$\left[\underbrace{(A^TA)^{-1}}_{\text{invertible}}A^T\right]\underset{m\times n}{A} = \underset{n\times n}{I}$$

  $n \times m$

  A^{-1}_{left}.

- right inverse [Full row rank]

  $r = m < n$

  $N(A^T) = \{0\}$ indep rows

  $\boxed{\infty \text{ Solutions to } Ax = b.}$ $n - m$ free variables.

  $$A\left[A^T(AA^T)^{-1}\right] = I$$

  right invers (A^{-1}_{right})

Left inv → $A\boxed{(A^TA)^{-1}A^T}$ → Projection onto col space.

Right inv → $\boxed{A^T(AA^T)^{-1}}A$ → projection onto row space

- pseudo-inverses

If $x \neq y$ in row space then $Ax \neq Ay$

Suppose $Ax = Ay$

$$\underbrace{A(x-y)}_{} = 0$$

↓

In nullspace, also in row space → not possible

$\therefore Ax \neq Ay$.

Find the pseudo inverse A^+

① start from SVD : $A = U\Sigma V^T$

U orthogonal

Σ diagonal $\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_r \end{pmatrix}$ m rows, n cols, rank = r

V orthogonal

Σ^+ pseudoinverse of diagonal : $\begin{bmatrix} 1/\sigma_1 & & 0 \\ & 1/\sigma_2 & \\ 0 & & 1/\sigma_r & 0 \end{bmatrix}$ $n \times m$

$\Rightarrow \Sigma\Sigma^+ = \begin{bmatrix} 1 \cdots & 0 \\ 0 & \boxed{0} \end{bmatrix}$ $(m \times m)$ projection onto col space

$\Sigma^+\Sigma = \begin{bmatrix} 1 \cdots & 0 \\ 0 & \boxed{0} \end{bmatrix}$ $(n \times n)$ projection onto row space

$$\boxed{A^+ = V\Sigma^+U^T}$$

이거 땡이 pseudo inverse!!